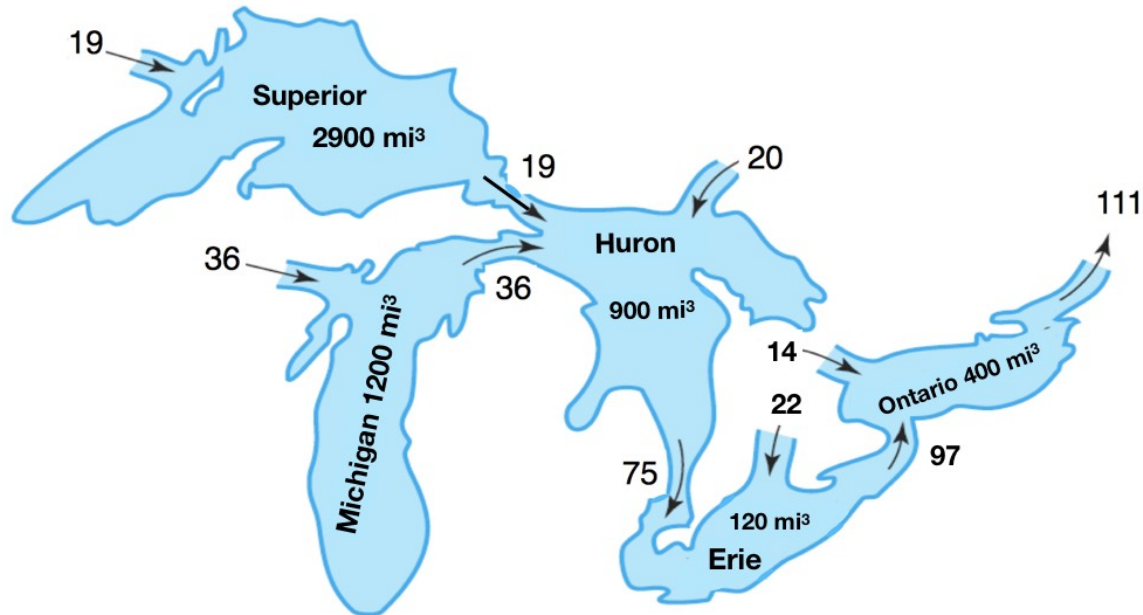


Math 2500 Project: Cleaning Up the Great Lakes

The Problem

A simple mathematical model that can be used to determine the time it would take to clean up the Great Lakes can be developed using a multiple compartmental analysis approach. In particular, we can view each lake as a tank that contains a liquid in which is dissolved a particular pollutant (DDT, phosphorus, mercury). Schematically, we view the lakes as consisting of five tanks connected as indicated in the figure.



For our model, we make the following assumptions:

- The volume of each lake remains constant.
- The flow rates are constant throughout the year. Flow rates in the figure are expressed in cubic miles per year.
- When a liquid enters the lake, perfect mixing occurs and the pollutants are uniformly distributed.
- Pollutants are dissolved in the water and enter or leave by inflow or outflow of solution.

Before using this model to obtain estimates on the cleanup times for the lakes, work through the two simpler models below. Include a short introduction to each scenario.

1. Use the outflow rates given in the figure to determine the time it would take to “drain” each lake. This gives a lower bound on how long it would take to remove all the pollutants.
2. A better estimate is obtained by assuming that each lake is a separate tank with only clean water flowing in. Use this approach to determine how long it would take the pollution level in each lake to be reduced to 50% of its original level. How long would it take to reduce the pollution to 5% of its original level?
3. Finally, to take into account the fact that pollution from one lake flows into the next lake in the chain, use the entire multiple compartment model given in the figure to determine a set of scalar equations that describe the amount of pollutant in each lake as a function of time. Assume the pollution has ceased (that is, inflows not from a lake are clean water) and that all the lakes initially have the same pollution concentration of 1 slug/mi³. Include a short introduction to each part.
 - i. Write the system of differential equations that models this system.
 - ii. Formulate a matrix equation for this system of equations.
 - iii. Using Matlab, solve the matrix equation for its eigenvalues, eigenvectors and constants and then translate these components of the solution to the matrix equation into a system of functions

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(scalar equations) that describe the pollutant level of each lake as a function of time. (Include your Matlab script in an appendix – not in the main body of your paper.)

- iv. Use a numeric solver in Matlab to determine when the pollution level in each lake has been reduced to 50% of its original level. (Do you need to use the numeric solver for all the pollution equations? Explain.) **Comment on whether or not your solutions make sense when compared to your simpler estimates.**
- v. Make similar determinations of the times required for the pollution to be reduced to 5% of its original level. **Comment as above.**
- vi. Plot the mass of pollutant vs. time for each lake. You should have two separate graphs, one for the 50% pollution reduction and one for the 95% reduction. Combine the plots for all the lakes in each graph.
- vii. Also, plot the concentration of pollutant vs. time, again, in two plots as described above.

Notes:

1. You may want to plot the functions earlier than indicated above in order to provide insight into what to expect for the equation solutions.

Guidelines for the Paper

This project will be a group effort with each group submitting a single paper. Your paper should have a professional appearance, including being typed (including the equations).

The paper describing your findings should include:

- (a) A short introduction describing the problem,
- (b) A section for each of the 3 steps of the problem outlined above. Describe the process by which you arrived at each of your solutions. While you should do your matrix calculations and numerical solutions in Matlab, show your starting equations (describing how they were developed), describe your approach to coding your work in Matlab, and show your equations for pollutant mass and concentration as well as the Matlab plots of these functions.
- (c) Citations of any references used.
- (d) Your Matlab code in an appendix. (Do not incorporate your code into the body of the paper and don't substitute code for your descriptions and equations.)

Your paper should provide a linear progression of thought and work understandable by someone "skilled in the art" without the need to reference outside sources including this problem statement.

Make sure your plots are labeled with a graph title, axes titles and scales and a legend identifying the various curves in each plot.

All members of your group should review the paper and approve it prior to its submission.

In separate documents, one per person, provide input on how well your group worked together and the relative contributions of your teammates. This document should be submitted through Blackboard.

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Milestones

Sunday, November 4	1. A written progress report (a single typed sheet) from your team captain (but complied by the entire group). The report should include: a. A list the members of your group, b. The defining differential equations, c. A <u>rough</u> draft of your code, d. Any questions you may have at this point.
Sunday, November 18	2. Submit the “contributions” documents on Blackboard

Matlab Coding Aids

A list of coding aids is posted separately.